CHAPTERWISE QUESTION

MATHEMATICS

SET A

DIFFERENTIAL EQUATIONS

CLASS - XII **Time : 1½ hrs.**

Marks : 40

SECTION - A **10 × 1 = 10**

1. The order of the differential equation $2x^2\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + y = 0$ is

   a) 2 b) 1 c) 0 d) Not defined

2. The degree of the differential equation $1 + \left(\frac{dy}{dx}\right)^2 = x$ is

   a) 1 b) 2 c) 3 d) Not defined

3. The integrating factor of the differential equation $x\frac{dy}{dx} - y = \log x$ is

   a) x b) $-\frac{1}{x}$ c) $\frac{1}{x}$ d) –x

4. The solution of $x\frac{dy}{dx} + y = e^x$ is

   a) $y = \frac{e^x}{x} + \frac{k}{x}$ b) $y = xe^x + kx$ c) $y = xe^x + k$ d) $x = \frac{e^y}{y} + \frac{k}{y}$

5. The number of arbitrary constants in the particular solution of a differential equation of second order is (are)

   a) 0 b) 1 c) 2 d) 3

6. Write the sum of the order and degree of the differential equation $\frac{d}{dx}\left\{\left(\frac{dy}{dx}\right)^3\right\} = 0$

   a) 4 b) 3 c) 2 d) 0

7. Write the sum of the order and degree of the differential equation $y = px + \sqrt{1 + p^2}$, where $p = \frac{dy}{dx}$

   a) Order 1, degree 2 b) Order 2, degree 4

   c) Order 1, degree 3 d) Order 2, degree 2

8. The solution of $\frac{dy}{dx} + y = e^{-x} + y(0) = 0$ is

   a) $y = x^{-1}$ b) $y = xe^{-x}$ c) $y = e^{x^2}$ d) $y = x^2e^{-x}$

9. The order and degree (if defined) of the differential equation $\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 = 2x^2 \log\left(\frac{d^2y}{dx^2}\right)$ are

a) 2 and 3 b) 2 and 1 c) 2 and not defined d) None of these

For question number 10 two statements are given - one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below.

a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

c) Assertion (A) is true but Reason (R) is false.

d) Assertion (A) is false but Reason (R) is true.

10. Assertion (A) : Degree of the differential equation $\left(\frac{dy}{dx}\right)^4 + 3y\frac{d^2y}{dx^2} = 0$ is 1

Reason (R) : Differential equation is a polynomial equation in derivatives and the highest power of the highest order derivative involved is the degree of the differential equation.

SECTION - B **2 × 2 = 4**

11. Find the general solution of the differential equation $\frac{dy}{dx} + \frac{1}{x} = \frac{e^y}{x}$

12. Write the integrating factor in the differential equation $(x \log x)\frac{dy}{dx} + y = 2 \log x$

OR

Solve $\frac{dy}{dx} + y = 1$.

SECTION - C **4 × 3 = 12**

13. Find a particular solution of the differential equation $\frac{dy}{dx} + y\cot x = 2x + x^2\cot x,\ x \neq 0,$ given y = 0 when $x = \frac{\pi}{2}$.

OR

Find the particular solution of the differential equation $\log\left(\frac{dy}{dx}\right) = 3x + 4y$ given that y = 0 when x = 0.

14. Solve the differential equation $\left[\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right]\frac{dx}{dy} = 1,\ x \neq 0$

15. Solve : $\frac{dy}{dx} = e^{x-y} + x^2e^{-y}$

16. Find a particular solution of the differential equation $\frac{dy}{dx} = 1 + x + y + xy,$ given y = 0 when x = 1.

SECTION - D **2 × 5 = 10**

17. Find the general solution of the differential equation $\frac{dy}{dx} - y = \cos x$.

18. Solve the differential equation $(1 + x^2)\frac{dy}{dx} + y = \tan^{-1} x$

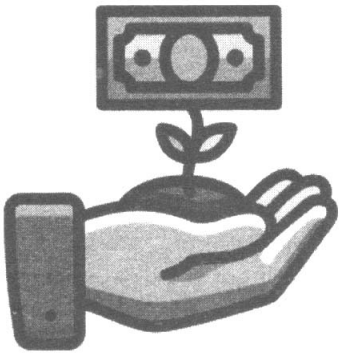
OR

In a bank principal increases at the rate of r% per year. Find the value of *r* if Rs. 100 double itself in 10 years (given log 2 = 0.6931).

SECTION - E

Case Study

19. It is known that, if the interest is compounded continuously, the principal changes at the rate equal to the product of the rate of bank interest per annum and the principal. Let *P* denotes the principal at any time *'t'* and rate of interest be *r*% per annum.



i) Find the value of $\frac{dP}{dt}$. **1**

ii) If P_0 be the initial principal, then find the solution of differential equation formed in given situation. **2**

iii) If the interest is compounded continuously at 5% annum, in how many years will ₹ 100 double itself? **1**

CHAPTERWISE QUESTION

MATHEMATICS

SET B

DIFFERENTIAL EQUATION

CLASS - XII **Time : 1½ hrs.**

Marks : 40

SECTION - A **10 × 1 = 10**

1. Find the product of the order and degree of the differential equation $x\left(\frac{d^2y}{dx^2}\right)^2+\left(\frac{dy}{dx}\right)^2+y^2=0$

   a) Order 4, degree 4 b) Order 1, degree 2

   c) Order 2, degree 2 d) Order 5, degree 2

2. Write the degree of the differential equation $x\left(\frac{d^2y}{dx^2}\right)^3+y\left(\frac{dy}{dx}\right)^4+x^3=0$

   a) Degree 3 b) Degree 4 c) Degree 2 d) Degree 1

3. The number of arbitrary constants in the particular solution of a differential equation of third order are .....

   a) 0 b) 2 c) 4 d) 1

4. The degree of the differential equation $\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}=\frac{d^2y}{dx^2}$ is

   a) 4 b) $\frac{3}{2}$ c) not defined d) 2

5. The integrating factor of $\frac{dy}{dx}+y=\frac{1+y}{x}$ is

   a) $\frac{e^x}{x}$ b) $\frac{e^{-x}}{x}$ c) e^x d) e^{-x}

6. The general solution of the differential equation $e^{2x}\frac{dy}{dx}+3e^{2x}y=1$ is

   a) $ye^{3x}=e^x+C$ b) $ye^{3x}=e^{-x}+C$ c) $ye^{3x}=-e^x+C$ d) $ye^x=e^{3x}+C$

7. The general solution of $\frac{dy}{dx}=\sqrt{4-y^2}$, where $-2<y<2$ is

   a) $y=\sin(x+C)$ b) $y=2\sin(x+C)$ c) $y=2\cos(x+C)$ d) $y=\cos(x+C)$

8. The integrating factor of $(\sin x)\frac{dy}{dx} + (2 \cos x)\, y = \sin x \cos x$ is

a) sec x b) $(\sin x)^2$ c) $(\text{cosec } x)^2$ d) $(\tan x)^2$

9. The general solution of $xy \frac{dy}{dx} = (x + 3)(y + 3)$ is equal to

a) $x - 3\log|x| + C$ b) $x + 3\log|x| + C$ c) $3x + \log|x| + C$ d) None of these

For question number 10 two statements are given - one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below.

a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

c) Assertion (A) is true but Reason (R) is false.

d) Assertion (A) is false but Reason (R) is true.

10. Assertion (A) : Order of the differential equation $\left(\frac{dy}{dx}\right) + \sin\left(\frac{dy}{dx}\right) = 0$ is 1

Reason (R) : Order of a differential equation is the order of the highest order derivative of the depended variable with respect to the independent variable involved.

SECTION - B **2 × 2 = 4**

11. Solve the initial value of problem $e^{\frac{dy}{dx}} = x + 1$, $y(0) = 5$.

12. Show that the differential equation given by is $x^2 \frac{dy}{dx} = x^2 - 2y^2 + xy$ is homogeneous.

OR

Solve the equation $x^5 \frac{dy}{dx} = -y^5$

SECTION - C **4 × 3 = 12**

13. Solve the differential equation $\sec^2 x \tan y\, dx + \sec^2 y \tan x\, dy = 0$.

14. Find a particular solution of the differential equation $x(x^2 - 1)\frac{dy}{dx} = 1$ given $y = 0$ when $x = 2$.

15. Solve the differential equation $(1 + x^2)\, dy + 2xy\, dx = \text{Cot}x\, dx$.

16. Find a particular solution of the differential equation $\left[x\text{Sin}^2\left(\frac{y}{x}\right) - y \right] dx + xdy = 0$, given $y = \frac{\pi}{4}$ when x = 1

OR

Find a particular solution of the differential equation $2xy + y^2 - 2x^2 \frac{dy}{dx} = 0$, given y = 2 when x = 1

SECTION - D **2 × 5 = 10**

17. Show that the differential equation $2ye^{\frac{x}{y}} dx + \left(y - 2xe^{\frac{x}{y}} \right) dy = 0$ homogeneous. Find the particular solution given that x = 0 when y = 1.

18. Solve : $\frac{dy}{dx} - 3y \cot x = \sin 2x$;. Find the particular solution when y = 2 when x = $\frac{\pi}{2}$.

OR

Find the particular solution of the differential equation $(\tan^{-1} y - x)\, dy = (1 + y^2)\, dx$, given that x = 0, y = 0.

SECTION - E

Case Study

19. In a college hostel accommodating 1000 students, one of the hostellers came in carrying Corona virus and the hostel was isolated. The rate at which the virus spreads is assumed to be proportional to the product of the number of infected students and remaining students. There are 50 infected students after 4 days.

i) If $n(t)$ denote the number of students infected by Corona virus at any time t, then find the maximum value of $n(t)$. **1**

ii) Find the value of n(4). **1**

iii) Find the most general solution of differential equation formed in given situation. **2**

OR

Find the value of n at any time.